**a. Rayleigh Fading (caution! not final, notes only!)**

To calculate the Secrecy Outage Probability for a given system model we need the channel coefficient and the Shannon-Hartley theorem. The channel coefficients are based on the probability density function of the channel’s distribution. The three channels that are going to be tested in this sample report are the Rayleigh channel, the Nakagami-m channel, and the Weibull channel.

The Rayleigh fading or Rayleigh channel is a statistical model for the effect of a propagation environment on a radio signal, such as that used by wireless devices. Rayleigh fading models assume that the magnitude of a signal that has passed through such a transmission medium will vary randomly, or fade, according to a Rayleigh distribution. This distribution is the radial component of the sum of two uncorrelated Gaussian random variables. Rayleigh fading is viewed as a reasonable model for tropospheric and ionospheric signal propagation as well as the effect of heavily built-up urban environments on radio signals. Rayleigh fading is most applicable when there is no dominant propagation along a line of sight between the transmitter and receiver. If there is a dominant line of sight, Rician fading may be more applicable.

**b. System Model using Rayleigh Fading Channel**

As foretold, the scatters model is a reasonable one when there are many objects in the environment that scatter the radio signal before it arrives at the receiver. The central limit theorem holds that, if there is sufficiently much scatter, the channel impulse response will be well-modelled as a Gaussian process irrespective of the distribution of the individual components. This means the impulse response varies based on time and the symbol delay. If there is no dominant component to the scatter, then such a process will have zero mean and phase evenly distributed between 0 and 2π radians. The envelope of the channel response will therefore be Rayleigh distributed.

Calling this random variable , it will have a pdf:

Where which is the second moment of the random variable, in other words it is called mean-squared value, which is the mean of its square and not the square of its mean. When the distribution is centered on zero, then the second moment is the variance of the random variable since:

As told, the distribution is zero-centered, which means that , thus . This means that:

Rayleigh fading is exhibited by the assumption that the real and imaginary parts of the response are modelled by independent and identically distributed zero-mean Gaussian processes so that the amplitude of the response is the sum of two such processes.

Based on [1] the GG distribution of the random variable R is given by:

In this equation the a is the fading parameter, c is the normalized variance of the channel envelope R, and is the ath mean square of the channel envelope. The gamma function is the following:

By changing the parameters of the GG distribution, we can obtain other famous distributions like Rayleigh, Rice, Weibull and Nakagami-m. Beginning the substitution using the Rayleigh parameters, where and .

Before returning to this equation we should calculate the gamma function:

Since , then:

The upper equation is the exact same as the Rayleigh model that we presented in relation (1). The , thus the final model equation is:

The problem must be approached from a channel perspective. The main metric for a channel is the capacity and the SNR, in other words the quality of the transmitted signal. Thus we are going to assume that in our earlier formulas, the main random variable corresponded to the SNR of the channel and not the actual scale. Doing that we are talking about the distribution of the SNR which varies with time. The general pdf of the SNR for a GG fading channel is given as:

In the upper formula, the parameter k is assigned as the Destination (D) or the Eavesdropper (E) channel. are the normalized variances of the two channel envelopes based on the bandwidth. The average SNR is defined as:

The ratio is the energy per bit to the noise power spectral density. By substituting the parameters for the Rayleigh fading (, and also assuming that the parameters are the same for both the main and eavesdropper channels, we have the following equation:

As for the cumulative density function, we have the following equation, which is based on the lower gamma function.

We know that the lower gamma function has the following expression:

Thus the previous equation becomes

The former can be solved very easily using integration by factors, resulting in the following final expression:

The upper formulas are valid when it is known that the channel is overcome by Rayleigh fading.

**d. Secrecy Outage Probability Analysis**

Secrecy Outage Probability is defined as the probability that the instantaneous secrecy capacity falls below a predesignated target bitrate. Simplifying the definition, this is the probability that the channel will seize being secure, and that the eavesdropper can discern critical information about the transmitted data. Thus, SOP is an important performance measurement, which is widely used to characterize a wireless communication system.

SOP can be defined as:

By considering that we are using bit transmission we have the capacity as it was defined by Shannon-Hartley. In the upper formula the describes the predesignated threshold capacity for the secrecy outage. Respectively, the describes the ratio of the destination capacity to eavesdropper capacity.

This capacity is normalized by the channel bandwidth. Thus, we have the following:

We will apply some simplifications on the previous expression by using the function which is 1-to-1, keeping the monotony of the function unchanged.

To calculate the probability, we need to calculate the area below the pdf up until the break point. This is achievable by using the cumulative distribution function:

We set , and then apply the first integral:

And then we also integrate for the second SNR, which is the eavesdropper’s channel.

We solve the inner integral by using the formula of the CDF, and we have the following:

Both the cumulative and the density functions are known from earlier calculations. We can also notice that the cumulative is irrespective of which allows us to place it on the outside of the integral. Then we will attempt to solve it:

Respectively, we have the following:

The final integral to be solved is the following:

We will now attempt to simplify the equation:

Our solution of the upper integral assumes that the big terms in the exponential powers will be simplified by substituting them with some placeholder variables.

Using this separation, we will execute the multiplication inside the integral

Thus, the integral will be transfigured as such:

We will solve each integral separately:

Substituting the solutions in the initial integral we have

Since we know that , then we can simulate the expression using various values of the threshold channel capacity and see how the SOP changes, responding to the capacity. Another simulation we can execute is by changing the parameter. The parameter is defined as the following ratio:

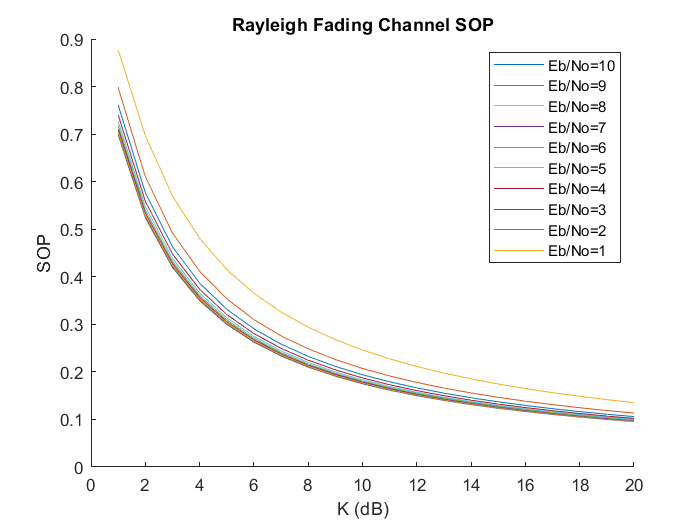
By defining the threshold capacity as 1 dB, the normalized variances of both the destination and eavesdropper receivers being and the , then the simulation yields the following results:

*Chart

Description automatically generated*

The K value varies from 20 down to 0 with step -1. As we can see, the Secrecy Outage is more likely to happen when K = 1, which means that the average SNRs for both destination and eavesdropper have the same value. The threshold capacity is 1, which means that, the case that both SNRs are equal is the Marginal Outage Probability, which is 68.29%. As the ratio between destination and eavesdropper widens, the secrecy outage probability diminishes. For a K equal to 1000, the sop is 0.2046%, which is significantly smaller.

In the next simulation we will also variate the , which is the noise variance. The noise variance is moving from 10 down to 1, since 0 is not an acceptable value in our simulation. The upper simulation yields the following results:

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We can see that the bigger the noise the lesser the Secrecy Outage Probability.

**e. System Model using Weibull Fading Channel**

This is the second channel fading we are going to experiment with. The Weibull distribution for a random variable R is defined as:

In the upper formula the k is the shape parameter and λ is the scale parameter of the distribution. We can interpolate between the exponential distribution using and the Rayleigh distribution with and . The shape parameter indicates the failure rate monotony. When k is negative, the failure rate diminishes over time, whereas for k positive, the failure rate increases over time.

In reference to the conducted empirical studies, this distribution can describe a wireless channel fading effective enough to model both indoor and outdoor environments.